A Net-Theoretic Approach for the Construction and Analysis of Logically Based Problem Descriptions[1]

By S. Zelewski[2]

Abstract: The construction and analysis of OR-problems are considered under the assumption that the problem structures depend essentially on logical constituents. The conventional approach to use binary ("0/1") variables suffers from models which are very complicated and intransparent. The nature of logical constituents is not "adequately" represented by complexes of binary variables. As an alternative the net-theoretic approach can be used to construct graphical problem descriptions. These descriptions base on the propositional or predicate logic and the theory of Petri nets. The resulting net models have the advantages of compactness and easy interpretation. Furthermore conventional algebraic algorithms can be applied to the analysis of net models. The efficiency of model construction and analysis is not considered. Instead of this the preceding aspect is explored which contributions to problem modelling result from combining logic, net theory, linear algebra and graphical problem description. Some examples demonstrate these results. They especially concern the design of annual balance sheets of stock corporations with regard to corporate income tax.

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1 This article is an abbreviated and revised version of the working paper Zelewski (1986), available from the author.

2 Dr. S. Zelewski, Universität Köln, Seminar für Allgemeine Betriebswirtschaftslehre, Industriebetriebslehre und Produktionswirtschaft ("Industrieseminar"), Albertus-Magnus-Platz, 5000 Köln 41, FRG.

1 Logically Based Problem Descriptions for Decision Models

Decision models, which formally represent real decision problems, are often designed as algebraic programs ("OR-programs"); see Bitz (1977), Laux (1982), Ellinger (1985), Kern (1987). Each program regularly possesses a multi-dimensional space of possible problem solutions. These problem space is a subspace of the cartesian product of all variables that occure in the program definition.

The search for an intended - e.g. optimal - problem solution can be efficiently realized as far as the problem space is dense (and convex). Therefore it is mostly tried to construct OR-programs with real-valued variables. The density-assumption for the problem space can be regarded as adequate if the variables represent real entities to be measured on metric scales.

Such a real-valued representation of problem constituents looses adequacy if logical aspects must be modelled. This holds at least for the classical propositional and predicate logic which allow only two distinct truth values. These both versions of logic are supposed in the following[3]. The logical aspects of a problem can be separated into two categories. The first category concerns subproblems of the yes/no-type that require decisions whether an action should be done ("yes") or not ("no"). The second category reflects logical dependencies between problem constituents, for example between partial decisions or between a decision and its real consequences.

Both categories of logical aspects are usually modelled in OR-programs with the help of binary logic variables x_i with domains $D_i = \{0;1\}$. They are called decision (indicator) variables if they represent subproblems of the yes/no-type (locical dependicies); see

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3 Real-valued logics that may be grounded on fuzzy set theory or theories of evidence are not considered.

Williams (1985). The resulting problem descriptions are mixed integer programs which suffer from a great structural and computational complexity.

The computational complexity grounds on the combinatorial explosion of possible problem solutions - with regard to a increasing number of problem describing binary logic variables - and the fact that techniques of differential calculus cannot be (directly) applied to mixed integer programs; see Forrest (1974), Gabriel (1982), Williams (1985). The quantitative aspect of computational complexity does not play any role in the following, because the efficiency of problem solution is beyond the scope of this explorative study. Furthermore the analysis of net models which will be presented later is subject to the same difficulty of combinatorial explosion, at least in the case of analyzing net invariants.

The structural complexity describes a qualitative characteristic of decison models based on binary logic variables. Such models are generally very extensive and complicated problem descriptions because of great numbers and multiple interlockings of logic variables; see for example Gabriel (1982), Williams (1985), Johänntgen-Holthoff (1986) and Boos (1986). Especially the representation of logical dependencies between decisions and their consequences often leads to intransparent comglomerates of decision, indicator and "normal" variables. The inherent complexity of the resulting decision models impedes validating and using the models. For example it is hard to justify a recommended decision alternative when the underlying decison model is so complicated that nobody understands the model - except der model designer itself.

For that reason there is a need for model constructing techniques which allow more transparent representations of logical aspects in decision models. One possible approach (among others) to overcome the lack of transparency of conventional OR-programs is the construction of graphical problem descriptions. They base on the experience that graphical models are compacter and easier to understand as the "variable-conglomerats" of OR-programs.

Graphical modelling techniques should fulfil two additional requirements. On the one hand they should make it possible to systematically derive the representation of logical problem aspects from a description of these aspects expressed in natural language (constructive requirement), because most real problems are firstly circumscribed with natural language statements. The derivation is designated as systematical if there exists a scheme which enables to derive representations for all logically expressable problem descriptions.

On the other hand it is desirable that the graphical models can be analyzed with the help of algebraic techniques which are computerized executable (analytical requirement). Such techniques are selected since they have widely been proved to be successful with respect to analytical purposes. The additional postulate of possible computerization reflects the practical point of view that analyses of real problems usually consume great amouts of information processing ressources. Therefore they can often realized only if they are automatically executed by software implementations of the analytical algorithms.

Two reasons motivate to discuss the modelling of logical aspects of real problems. Firstly OR-models depend on such logical aspects in many ways, even if their logical nature does not always appear explicitly. Examples for logical aspects are priorities for sequencing and scheduling of jobs, requirements of exclusiveness and completeness in allocation models (like the allocation of warehouses) or the existence of fixed capacity costs which can be eliminated in the case of producing nothing; see for more examples Williams (1985).

Secondly an increasing number of publications reflects the combination of operations research and artificial intelligence; see Bullers (1980), Müller-Merbach (1984), Thornton (1985) and Neumann (1987). The logically based representation of knowledge is one of the most important concepts for the description of problems which have been developed by artificial intelligence researchers. Therefore models with explicitely represented logical aspects will play a greater role in future OR-applications. Especially it is intended to integrate algorithms that are wellknown for the solution of special OR-models into expert systems. Such expert systems shall serve as intelligent models and methods storage bank systems in order to assist their users in solving a wide range of problems. Even outside the context of artificial intelligence problem descriptions with explicit representation of logical aspects gain increasing attention for business applications. In Bonczek (1981) e.g. the design of decision support systems is described which ground on predicate logic fomulas for the construction of decision models.

2 Construction of Net Models

2.1 Net-Theoretic Foundation

The theory of Petri nets offers one way to construct graphical models which are able to represent logical aspects of the underlying real problems and fulfil the two forementioned constructive and analytical requirements. An additional reason to concentrate on Petri nets is the fact that these nets are often used for logically based knowledge representations; see Zisman (1978), Azema (1984), Mainz (1984), Giordana (1984), Fidelak (1986a) and Fidelak (1986b). This corresponds with the trend to combine operations research and artificial intelligence stated above.

For the purpose of simplicity the following arguments refer only to logical aspects which can be formally expressed as formulas of propositional logic[4]. Under this assumption it is sufficient to deal only with place/transition-nets which are a simple and easily intelligible version of Petri nets[5]. A place/transition-net is a 6-tupel $N=(P,T,F,K,W,M_0)$ for which the following definitions hold:

- $P=\{p_j \mid j=1,...,J\}$ is a finite set of atomar objects which are designated as places and graphically represented by circles. Places may be marked by an arbitrary number of tokens which are movable, undistinguishable atomar objects.
- $T=\{t_i \mid i=1,...,I\}$ is a finite set of atomar objects which are designated as transitions and graphically represented by rectangles, usually squares. Transitions can change the distribution of tokens over places by removing and dropping tokens. Each such change is called the occurence or firing of a transition.
- $F \subseteq ((PxT) \cup (TxP))$ is the flow relation. Each element of this relation is an ordered pair (p_j,t_i) or (t_i,p_j) which expresses a flow of tokens removed from or dropped to place p_j by firing transition t_i.

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4 See Esser (1977) and Stegmüller (1983) for precise explanations of the calculus of propositional logic. The concentration on propositional logic is admissible because most logical aspects of OR-problems can be expressed with the help of propositions; see Williams (1985) and Johänntgen-Holthoff (1986). One of the rare exceptions is the description of a flexible manufacturing system by Bullers (1980), which is based on predicates. But this model originates from an aritificial intelligence context, not from a operations research study. See Lautenbach (1984), Mainz (1984) and Fidelak (1986a) for the special difficulties concerning the computational complexity which arises when the structure of net models shifts from propositional to predicative formulas.

5 See Jantzen (1980) and Reisig (1987) as introductions in the theory of place/transition-nets.

The element of the flow relation is graphically represented by an edge directed from the (circle for) place p_j to the (square for) transition t_i or from the transition t_i to the place p_j, respectively.

- $K: P \rightarrow N_+ \cup \{\infty\}$ is the capacity function which assigns a capacity $K(p_j)$ of tokens to each place p_j[6]. The capacity $K(p_j)=\infty$ is chosen if there does not exist any finite limitation for the number of tokens on a place p_j.
- $W: (P\times T) \cup (T\times P) \rightarrow N_0$ is the weight function which assigns a weight $W(x_a, x_b)$ to each ordered pair (x_a, x_b) consisting of a place and a transition[7]. If the pair (x_a, x_b) is an element of the flow relation F the weight $W(x_a, x_b)$ must be positive, otherwise it equals zero.
- $M_0: P \rightarrow N_0$ is the initial marking function of the net in its original state. It assigns an arbitrary, but finite number $M_0(p_j)$ of tokens to each place p_j which holds as long as no transition has been fired. $\underline{M}_0=(M_0(p_j) \mid j=1,...,J)^{tr}$ is the marking vector of the net in its original state and is designated as initial marking.

The graphical representation of a place/transition-net $N=(P,T,F,K,W,M_0)$ is a bipartite, directed, inscribed graph. The set $X=P\cup T$ of nodes is bipartite because nodes of place- and transition-type can be distinguished. The flow relation F is identical with the set of directed edges. The partial tupel $TOP_N=(P,T,F)$ which contains only the nodes and edges of the graphical net representation is designated as the topological net structure.

The graphical net represenation is enlarged by inscriptions. Nodes p_j, which represent places, are inscribed with capacities $K(p_j)$ and with $M_0(p_j)$ dots for tokens that belong to the places under the initial

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6 N_+ is the set of all natural numbers without zero.

7 N_0 is the set of all natural numbers including zero.

marking $\underline{M}_0$. Edges (x_a, x_b) are inscribed with the weights $W(x_a, x_b)$. Nodes x_a and x_b which are connected by an edge are called incident, the corresponding edge (x_a, x_b) or (x_b, x_a) is designated as adjacent to node x_a and x_b. Each place p_j which is connected with the transition t_i by an edge (p_j, t_i) or (t_i, p_j) is called an input- or output-place of the transition, respectively. $pre(t_i)=\{p_j \varepsilon P | (p_j, t_i) \varepsilon F\}$ and $post(t_i)=\{p_j \varepsilon P | (t_i, p_j) \varepsilon F\}$ are the sets of all input- and output-places of transition t_i, respectively.

For the topology of place/transition-nets hold three premises which ensure that places and transitions are well distinguished objects, empty nets do not exist and there are no isolated nodes of place- or transition-type:

$$P \cap T = \emptyset$$

$$P \cup T \neq \emptyset$$

$$P \cup T = dom(F) \cup cod(F)$$

with: $dom(F)=\{x_a \varepsilon (P \cup T) | \vee (x_b \varepsilon (P \cup T): (x_a, x_b) \varepsilon F\}$

with: $cod(F)=\{x_b \varepsilon (P \cup T) | \vee (x_a \varepsilon (P \cup T): (x_a, x_b) \varepsilon F\}$

In the following two further assumptions hold. Firstly all places $p_j \varepsilon P$ possess the same token capacity $K(p_j)=1$. Therefore it is not necessary to inscribe places with capacities. All places p_j can only be marked in two admissible manners. Either they are marked according to $M(p_j)=1$ or they are unmarked according to $M(p_j)=0$. Secondly all edges $(x_a, x_b) \varepsilon F$ have the same weight $W(x_a, x_b)=1$, so that inscriptions of edges with weights may be neglected. Therefore the weight function can be simplified to:

$$W: (PxT) \cup (TxP) \rightarrow \{0;1\}$$

$$(x_a, x_b) \rightarrow W(x_a, x_b) = \begin{cases} 1; \text{ if } (x_a, x_b) \varepsilon F \\ 0; \text{ if } (x_a, x_b) \notin F \end{cases}$$

The static net structure is condensed in the incidence matrix $\underline{C}$ with I columns for the transitions $t_i \varepsilon T$ and J rows for the places $p_j \varepsilon P$. Each component $c_{i.j}$ indicates whether transition t_i is connected with place p_j and denotes - if they are incident - the direction of their connection[8]:

$$c_{i.j} = \begin{cases} W(t_i,p_j)=1 & ;\ \text{if } (t_i,p_j)\varepsilon F \wedge (p_j,t_i)\notin F \\ W(t_i,p_j)-W(p_j,t_i)=1-1=0; & \text{if } (t_i,p_j)\varepsilon F \wedge (p_j,t_i)\varepsilon F \\ 0; & ;\ \text{if } (t_i,p_j)\notin F \wedge (p_j,t_i)\notin F \\ -W(p_j,t_i)=-1; & ;\ \text{if } (t_i,p_j)\notin F \wedge (p_j,t_i)\varepsilon F \end{cases}$$

The dynamic net structure consists of two parts. The first part is given by the initial marking $\underline{M}_0$ which constitutes a boundary condition for all admissible evolutions of the net behavior. The second part is a firing rule which is not explicitely defined in the 6-tupel mentioned above. But the firing rule establishes the main difference between net theory and conventional graph theory. It enables to produce a new marking by firing a single transition - or a set of transitions[9] - under a given marking, so that a net may be regarded as a family of graphs with identical topologies, but varying markings.

Some definitions are required for the formal specification of the firing rule. All involved markings are defined like the initial marking $\underline{M}_0$ as vektors $\underline{M}=(M(p_j)\,|\,j=1,..,J)^{tr}$ with underlying marking functions $M: P \rightarrow N_0$. A firing vector $\underline{t}_i=(c_x\,|\,x=1,...,I)^{tr}$ is associated with the firing of a transition t_i in such a manner that for all components c_x of the vector holds

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8 The coefficients $c_{i.j}$ cannot distinguish between the case of no connection - i.e. $(t_i,p_j)\notin F$ and $(p_j,t_i)\notin F$ - and the case of 1-loops which are defined by the simultaneous holding of $(t_i,p_j)\varepsilon F$ and $(p_j,t_i)\varepsilon F$, since $c_{i.j}=0$ holds in both cases. The two edges of 1-loops are artificially eliminated by constructing incidence matrices. Therefore 1-loops cannot adequately treated by incidence matrices. But the special problems that may be caused by 1-loops need not be considered in this article because the constructed nets contain no such loops (so-called "pure" nets) as far as they do not express tautologies; see chapter 2.2.

9 The concurrently firing of several transitions is a special feature of Petri net theory which enables the modelling of non-sequential processes. It is neglected in the following, because it does not play any role with regard to the representation of logical problem aspects.

$c_x=1$ iff $x=i$ and $c_x=0$ iff $x \neq i$. The predicate $AKT(t_i, \underline{M})$ is valid iff transition t_i is activated under marking $\underline{M}$, i.e. iff the firing of transition t_i would not cause forbidden markings. Such markings would occure if the firing of the transition would lead to negative token "numbers" on its input places by removing tokens or would exceed the token capacity of its output places by dropping tokens. Therefore the activation of a transition t_i under a marking $\underline{M}$ is definied by:

$$AKT(t_i, \underline{M}) :\Leftrightarrow \ldots$$
$$(\bigwedge(p_j \varepsilon pre(t_i)): \; M(p_j) \geq W(p_j, t_i))$$
$$\wedge \; (\bigwedge(p_j \varepsilon post(t_i)): \; M(p_j)+W(t_i, p_j)-W(p_j, t_i) \leq K(p_j))$$

Regarding the forementioned assumptions for the capacity function K and the weight function W the definition of predicate $AKT(t_i, \underline{M})$ can be reduced to[10]:

$$AKT(t_i, \underline{M}) :\Leftrightarrow \ldots$$
$$(\bigwedge(p_j \varepsilon (pre(t_i)-post(t_i))): \; M(p_j)=1)$$
$$\wedge \; (\bigwedge(p_j \varepsilon (post(t_i)-pre(t_i))): \; M(p_j)=0)$$
$$\wedge \; (\bigwedge(p_j \varepsilon (pre(t_i) \cap post(t_i))): \; M(p_j)=1)$$

Now the firing rule can be formally specified as a partial function FR for which holds:

$$FR: \; T \times N_0{}^J \rightarrow N_0{}^J$$
$$(t_i, \underline{M}) \rightarrow \underline{M}'=\underline{M}+\underline{C} \cdot \underline{t_i}\,; \text{ if } AKT(t_i, \underline{M})$$

Under each marking $\underline{M}$ an activated transition may but need not be fired. Transitions which are not activated must not be fired.

An admissible behavior of a net consists of a sequence of transition firings starting from the initial marking and obeying the firing rule. Such a firing sequence is notated by a tupel $FS_q=(t_{i(1)}, t_{i(2)}, \ldots, t_{i(z)})$. During the execution of the sequence occure Z firings of - not necessarily distinguished - transi-

10 In the case of an 1-loop with $(p_j, t_i) \varepsilon F$ and $(t_i, p_j) \varepsilon F$ it is assumed that the involved transition t_i is activated iff the incident place p_j possesses exact one token. This assumption is not necessary but often used in net theory.

tions $t_{i(z)}$ with $z=1,\ldots,Z$[11]. The firing vector $\underline{t}_q=(c_x \mid x=1,\ldots,I)$ counts in each of its components c_x how often the corresponding transitions t_i with $i=x$ are fired in the firing sequence FS_q. If a firing sequence FS_q starts under a marking $\underline{M}$ – for example the initial marking $\underline{M}_0$ – and leads to a new marking $\underline{M}'$, it follows from the recursive application of the firing rule: $\underline{M}'=\underline{M}+\underline{C}\cdot\underline{t}_q$.

A marking $\underline{M}$ is designated as reachable with respect to a given initial marking $\underline{M}_0$ iff it exists at least one firing sequence which starts under the initial marking $\underline{M}_0$ and leads to the marking $\underline{M}$. A transition which is not activated under any reachable marking is called a dead transition or fact. A net whose transitions are all dead under the initial marking $\underline{M}_0$ is a fact-net.

2.2 Construction of Net Models Representing Logical Problem Aspects

The place/transition-nets introduced above allow to systematically transform any description of logical problem aspects into an equivalent net-theoretic representation[12]. The only premise which must be assumed for

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11 In this article only finite firing sequences are considered so that $Z \in N_+$ holds. But infinite firing sequence are also admissible in net theory.

12 See Thieler-Mevissen (1975), Thieler-Mevissen (1977) and Lautenbach (1985) for the underlying construction ideas.

this construction of net models is that the logical problem description grounds on propositional logic[13].

The construction scheme rests on the possibility to compose every proposition in a recursive manner with the help of some atomar propositions and propositional operators. Although the consideration of the operators for conjunction (or alternatively: adjunction) and negation is sufficient in order to compose any arbitrarily complex proposition, the operators for conjunction ("and"), adjunction ("or" in its inclusive sense), disjunction ("or" in its exclusive sense) subjunction ("if..., then...") and negation ("not") are examined. This enables a direct and easy transformation of statements expressed in natural language into components of a corresponding net model.

Each atomar proposition P_j is represented by an atomar net N_j with the topological structure $TOP_j = (\{p_j\},\{t_i\},\{(t_i,p_j)\})$. The characteristic component of this structure is the place p_j which is marked with one token iff the proposition is true[14]. Therefore the transition t_i must be dead iff proposition P_j is true under all reachable markings.

The negation $\neg P_j$ of an atomar proposition P_j is represented by a complementary atomar net $N_{\neg j}$ which differs from the net N_j only relating to an inversely directed edge. Therefore it possesses the topological

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13 The following arguments and constructions can be extended to descriptions of logical problem aspects based on predicate logic (of first order) by replacing place/transition-nets through predicate/transition-nets. But this extension leads only to a complication, but not to fundamentally new insights. Therefore it is neglected in the following. See Lautenbach (1985), Fidelak (1986a), Fidelak (1986b) and Zelewski (1986) for more detailed discussions of possible extensions to predicate logic and reductions of predicative problem descriptions to propositional descriptions.

14 On the contrary, the place p_j is unmarked - i.e. $M(p_j)=0$ - iff the corresponding proposition P_j is false; for only the markings $M(p_j)=1$ or $M(p_j)=0$ are admissible under the above introduced assumption of token capacities $K(p_j)=1$ for all places $p_j \varepsilon P$.

structure $TOP_{\neg j}=(\{p_j\},\{t_i\},\{(p_j,t_i)\})$. The characteristic place p_j is unmarked iff the proposition $\neg P_j$ is true, i.e. iff the underlying proposition P_j is false. For that reason the transition t_i is dead iff the proposition $\neg P_j$ is true for all reachable markings.

In the net-theoretic representaions of both atomar propositions P_j and negations $\neg P_j$ of atomar propositions P_j the characteristic places p_j substitute the propositions P_j. Therefore it can be stated in a simplified sense that each place p_j "represents" an atomar proposition P_j. The (un)marking of this place indicates that the corresponding atomar proposition P_j is true (false). The incident transition t_i is a fact iff the proposition P_j or $\neg P_j$ which is represented by the atomar net N_j or $N_{\neg j}$ (respectively) is true for all reachable markings[15].

The adjunction $P_j \vee P_k$ of two atomar propositions P_j and P_k is represented by the union of the two corresponding nets N_j and N_k, respectively. The union is constructed by identifying the two transitions of both nets as the same transition t_i. The places p_j and p_k, which are characteristic for the propositions P_j and P_k, are common output-places of the transition t_i. Therefore it follows for the topological structure $TOP_{j \vee k}$ of the composed net $N_{j \vee k}$:

$$TOP_{j \vee k} = (\{p_j, p_k\}, \{t_i\}, \{(t_i, p_j), (t_i, p_k)\})$$

If the adjunction contains the negation of an atomar proposition the corresponding characteristic place is the input-place of the common transition t_i. Therefore the adjunction $\neg P_j \vee P_k$ is represented by the net $N_{\neg j \vee k}$ with the topological structure:

$$TOP_{\neg j \vee k} = (\{p_j, p_k\}, \{t_i\}, \{(p_j, t_i), (t_i, p_k)\})$$

15 Because of the monotonicity of propositional logic a proposition must remain true if it has once been proved to be true. Therefore the formulation "... for all reachable markings" can be omitted for all net representations of consistent problem descriptions.

The net-theoretic representations of the adjunctions $P_j \vee \neg P_k$ and $\neg P_j \vee \neg P_k$ are constructed in an analogous manner. The constructions for adjunctions of more than two atomar propositions or their negations obey the same scheme. In each case the places which are characteristic for the involved atomar propositions are output- or input-places of a transition which is unique for the whole adjunction.

A tautology is a logically true proposition, i.e. a proposition that is true under all possible circumstances. Every tautology can be reduced to a proposition of the type $P_j \vee (\neg P_j)$. Following the representation technique for adjunctions of atomar propositions this tautology is represented by a net $N_{j \vee \neg j}$, which consists of an 1-loop. The topological structure of such an 1-loop is given by:

$$TOP_{j \vee \neg j} = (\{p_j\}, \{t_i\}, \{(t_i, p_j), (p_j, t_i)\})$$

Tautologies are logical constructs that do not represent any empirically meaningful knowledge. Therefore tautologies are neglected as components of logical descriptions of real problems in the following. The nets without 1-loops constitute the class of "pure" nets.

The disjunction $P_j \vee P_k$ of two atomar propositions P_j and P_k is represented by the union of the corresponding atomar nets N_j and N_k, respectively. But this kind of union differs from the construction explained before with regard to adjunctions of atomar propositions. The transitions of the both nets N_j and N_k must be distinct, so that they get the indices i and h with $i \neq h$. The resulting composed net $N_{j \vee k}$ has the topological structure:

$$TOP_{j \vee k} = (\{p_j, p_k\}, \{t_i, t_h\}, \ldots$$
$$\{(t_i, p_j), (t_i, p_k), (p_j, t_h), (p_k, t_h\})$$

A third kind of net union is required for the representation of the conjunction $P_j \wedge P_k$ of two atomar propositions P_j and P_k. The corresponding atomar nets N_j and N_k, respectively, possess once again distinct transitions t_i and t_h, respectively. But there are only two edges defined in the composed net $N_{j\wedge k}$, whose topological structure is given by:

$$TOP_{j\wedge k} = (\{p_j, p_k\}, \{t_i, t_h\}, \{(t_i, p_j), (t_h, p_k)\})$$

The net $N_{j\wedge k}$ consists of the two disconnected subnets N_j and N_k, because edges are missing which could connect the distinct transitions. The cases of negated atomar propositions and of more than two involved atomar propositions are treated analogously to the constructions for adjunctions of propositions.

The subjunction $P_j \rightarrow P_k$ of two atomar propositions P_j and P_k is represented by another union of the corresponding atomar nets N_j and N_k, respectively. Once again the transitions of both atomar nets are identified as a transition t_i. The characteristic places p_j and p_k become the input- and the output-place of this transition, respectively. The resulting composed net $N_{j\rightarrow k}$ possesses the topological structure[16]:

$$TOP_{j\rightarrow k} = (\{p_j, p_k\}, \{t_i\}, \{(p_j, t_i), (t_i, p_k)\})$$

All composed propositions which are complexer than the propositions discussed above can be reduced to those simple propositions with the help of inference rules for the substitution of logically equivalent propositions[17]. But it is easier to generalize the preceding constructions to the representation of conjunctive connected clauses.

The base for this generalization is the logical theorem that every finite proposition P - with arbitrary complexity - can be equivalently expressed in the conjunctive normal form; see Chang (1973). All logical aspects of a finite problem description can be transformed into such a conjunctive normal form. A finite

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16 This construction follows immediately from the representations of negations and adjunctions explained above; for it holds the equivalence: $P_j \rightarrow P_k \Leftrightarrow (\neg P_j) \vee P_k$.

17 The rules of de Morgan are examples for such inference rules: $\neg(P_j \wedge P_k) \Leftrightarrow (\neg P_j) \vee (\neg P_k)$ and $\neg(P_j \vee P_k) \Leftrightarrow (\neg P_j) \wedge (\neg P_k)$

proposition P fulfils the conjunctive normal form if it is a conjunction of clauses C_i: $P \Leftrightarrow C_1 \wedge \ldots \wedge C_I$ with $I \varepsilon N_+$. Each clause C_i (with $i=1,\ldots,I$) is either atomar or a adjunction of pairwise distinct literals $L_{i.r}$: $C_i \Leftrightarrow L_{i.1} \vee \ldots \vee L_{i.R_i}$ with $R_i \varepsilon N_+$. Each literal $L_{i.r}$ (with $r=1,\ldots,R_i$) is an atomar clause, i.e. either an atomar proposition P_j or the negation $\neg P_j$ of this atomar proposition[18]. Therefore any logical problem description P based on propositional logic can be represented with the help of atomar propositions P_j, clauses and conjunctions of clauses. The net N_P which represents the complex propositional problem description P is constructed as follows:

- establish for each clause C_i of proposition P a transition t_i;
- establish for each atomar proposition P_j a place p_j if this proposition is contained in at least one literal $L_{i.r}$ of at least one clause C_i;
- connect a transition t_i with a place p_j by an edge which is directed from the transition (place) to the place (transition) iff the corresponding literal $L_{i.r}$ is defined as an atomar proposition $L_{i.r}=P_j$ (as a negated atomar proposition $L_{i.r}=\neg P_j$).

For each clause C_i it results - as a generalization of the constructions for negations and adjunctions of atomar propositions - a subnet N_i with one transition t_i and R_i distinct places p_j. The topological structure TOP_i of this subnet is defined by:

$$TOP_i = (\{p_{j(1)},\ldots,p_{j(R_i)}\},\{t_i\}, \ldots \{ed(p_{j(1)},t_i),\ldots,ed(p_{j(R_i)},t_i)\})$$

with:

$$ed(p_{j(r)},t_i) = \begin{cases} (t_i,p_j); & \text{if } L_{i.r}=P_j \text{ is a literal of } C_i \\ (p_j,t_i); & \text{if } L_{i.r}=\neg P_j \text{ is a literal of } C_i \end{cases}$$

for all $r=1,\ldots,R_i$

18 The same atomar proposition must not be contained in more than one literal of the same clause; but it may be the component of several literals which belong to different clauses.

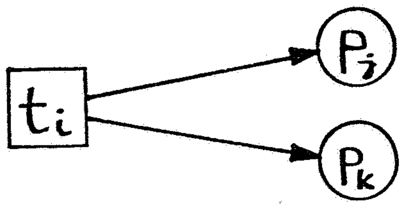
The complex proposition P, which is a conjunction of the I clauses C_i, is represented by the net N_P. This net results from the union of the subnets N_i corresponding to the clauses C_i. The union of subnets is constructed in the same manner as the union of atomar nets for the conjunction of atomar propositions. The resulting net is usually connected because places p_j which are characteristic for atomar propositions P_j are often contained in several clausel representing subnets. These common places are identified with eachother so that they integrate the subnets.

Fig. 1 summarizes all net-theoretic constructions for the representation of atomar propositions, for the negation, adjunction, disjunction, conjunction and subjunction of atomar propositions and for the generalized case of clauses.

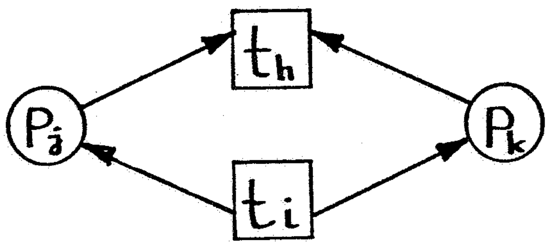
atomar proposition P_j

negation of the
atomar proposition P_j

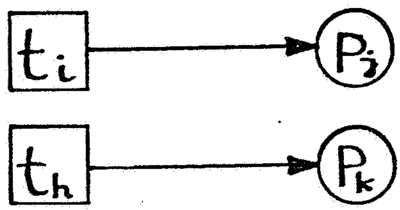
adjunction of the atomar
propositions P_j and P_k



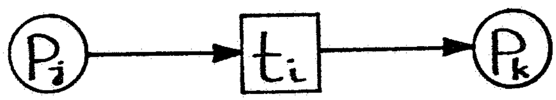
disjunction of the atomar
propositions P_j and P_k



conjunction of the atomar
propositions P_j and P_k



subjunction of the atomar
propositions P_j and P_k



clause C_i composed by
atomar propositions
P_1, P_2, P_3 and P_4:
$C_i \Leftrightarrow (\neg P_1) \vee (\neg P_2) \vee P_3 \vee P_4$

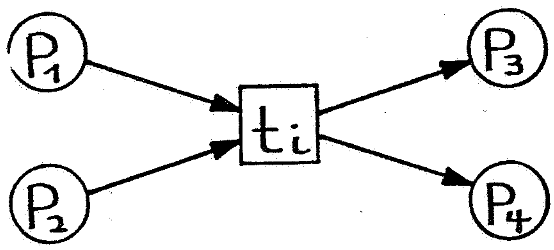


Fig. 1: net-theoretic constructions for the modelling
of logical problem aspects

The systematical construction technique which is based on clauses and their conjunctions fulfils the constructive requirement. The construction scheme covers all possible logical problem descriptions, because it holds for any proposition in conjunctive normal form and every proposition about logical problem aspects can be expressed in this normal form. The net N_P that represents such a problem describing proposition P is the intended net model of the logical problem aspects.

The construction of a net model N_P may be regarded as "natural". Problem descriptions in natural language can usually be reduced to elementary logical structures which stem from propositional logic. It is possible to transform such propositions systematically and directly into net-theoretic representations. No artificial logic variables, which often represent such logical problem aspects in an indirect and complex manner, are necessary; see for the usage of those variables in conventional OR-programs e.g. Boos (1986).

The natural representation of logical problem aspects especially holds if they are described with "If..., then..."-statements. Such statements are widespread both in business contexts as decison rules - for example as components of decision tables - and in artificial intelligence applications as production rules for ruled-based knowledge representation. Decision and production rules are - if they are restricted to propositional logic - subjunctions of atomar or composed propositions and can always be transformed into equivalent clauses. Therefore the generalized construction scheme based on conjunctions of clauses is a straightforward technique to derive net models from rule-oriented problem descriptions.

3 Approaches for the Analysis of Net Models

3.1 Analysis on the Ground of Fact-Nets

The construction of net models explained above leads to fact-nets, in which each transition - now called a "fact" - is not allowed to be fired, i.e. should be dead under every reachable marking; see Thieler-Mevissen (1977) for such fact-nets. Therefore a net model with a concrete marking of its places corresponds with a consistent description of logical problem aspects iff no transition is activated under that marking.

Otherwise - if at least one transition is activated and can be fired - there must be a logical contradiction in the problem description. The activated transition indicates which clause and which involved atomar propositions of the problem description cause the contradiction. The detection of such inconsistencies is very simple. For each transition of the net the activation-predicate AKT(t_i,$\underline{M}$) must be proved to be valid. It was shown earlier that this prove can be reduced to the algebraic test if the actual marking of the net fulfils two simply structured equality systems. If it holds[19]:

$$(\bigwedge(p_j \varepsilon pre(t_i)): M(p_j)=1) \wedge (\bigwedge(p_j \varepsilon post(t_i)): M(p_j)=0)$$

there must be a contradiction in the modelled problem description, for the "fact" t_i is inadmissibly activated. This inconsistency is caused by the assumption that all propositions P_j associated with the input-places p_j are true and all propositions P_j associated with the output-places p_j of transition t_i are false. This assumption is implied by the marking of the input-

19 For the two equality systems pure nets representing no tautologies are supposed. If impure nets shall be considered, too, the following equality systems have to be checked:

$$(\bigwedge(p_j \varepsilon (pre(t_i)-post(t_i))): M(p_j)=1)$$
$$\wedge (\bigwedge(p_j \varepsilon (post(t_i)-pre(t_i))): M(p_j)=0)$$
$$\wedge (\bigwedge(p_j \varepsilon (pre(t_i) \cap post(t_i))): M(p_j)=1)$$

and output-places with one or zero token(s), respectively, which activates the transition t_i.

The main advantage of net models concerns descriptions of real problems which are so voluminous that it is very hard to keep control over the implications of all logical problem constituents. In such cases compact and transparent net models can be constructed in order to apply computerized, simply structured algebraic algorithms for automatically detecting contradictions. In chapter 4.2 an example for detecting an inconsistency in the context of a balance sheet design problem is outlined.

The detection of inconsistencies in fact-nets grounds on the net-theoretic concept of markings. Each marking represents a meta-proposition about the truth of those atomar (object-)propositions which constitute a logical problem description. This meta-proposition possesses a situative quality, because its validity may vary in accordance to actual problem situations. The problem situation is defined by the assignment of truth values to all problem describing atomar (object-)propositions. The lastmentioned propositions themselves and their logical connections form the structure of a logical problem description[20].

Therefore a detected contradiction can rest on two different causes. Either it is grounded on an inconsistently described problem situation or its base is a structurally inconsistent problem description. In the fist case the contradiction can be resolved by changing the assignment of truth values, i.e. by varying the

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20 Correspondingly to this distinction a description of logical problem aspects is called structural iff it refers only to atomar propositions and their connections in composed propositions but does not reflect the possible truth values of those propositions. On the contrary, such a description is denotated as situative iff it is grounded on a structural description and contains an assignment of truth values to all atomar propositions. The truth values of the composed propositions can be determined with the help of truth tables which are defined for each propositional operator.

marking of the net model in such a manner that no more transition remains being activated. With the help of the net model as a transparent description of logical problem aspects it must be analyzed whether the formally resolved contradiction agrees with the real problem. If this can be shown then only the description of the actual problem situation was wrong.

Otherwise - if there exist no consistent marking variations or if the formal marking variations do not agree with real problem situations - the second case of a structurally inconsistent problem description occurs. The analysis of fact-nets does not cover difficulties of this kind, because it is marking-dependent. On the contrary, structural analyses must abstract from the actual problem situation which is expressed by meta-propositions about truth values and is represented in nets by markings. An net-theoretic approach that enables such an abstraction is the invariant analysis explained in the following.

3.2 Analysis on the Ground of Invariants

The invariant analysis of nets allows the detecting and exploration of structural inconsistencies in net-based logical problem descriptions. It neglects every concrete marking of a net, since it holds for any reachable marking. Only pure nets without 1-loops are analyzed[21]. Furthermore a special form of invariant analysis is considered in order to apply Lautenbach's net theorem. In this context two additional assumptions are required. Firstly it is abstracted from finite token capacities. Secondly initially unmarked nets with the

21 This assumption was already justified in chapter 2.2 by excluding tautologies. It is required because invariant analyses ground on incidence matrices $\underline{C}$, which cannot adequately cover 1-loops.

zero-marking $\underline{M}_0=\underline{0}$ - i.e. with $M_0(p_j)=0$ for all $p_j \varepsilon P$ - are supposed.

Invariants of place/transition-nets are defined with simple algebraic equalities; see Lautenbach (1987). They refer only to the incidence matrix $\underline{C}$ of nets. Therefore they describe structural aspects of the problems whose logical constituents are modelled with the help of nets.

Invariants are definied as special kinds of T- and P-vectors. A T-vector is an I-dimensional column vector $\underline{t}$. Its components c_x fulfil the requirements $c_x \varepsilon Z$[22], $x=1,\ldots,I$ and $I=\#(T)$ and correspond to transitions t_i with identical indices $i=x$. Accordingly a P-vector is a J-dimensional, integer column vector $\underline{p}$, whose components c_y with $c_y \varepsilon Z$, $y=1,\ldots,J$ and $J=\#(P)$ correspond to places p_j with identical indices $j=y$. A T- or P-vector is called semi-positive iff none of its components is negative and at least one component is greater than zero. A vector is called trivial iff each of its components equals zero.

A T-invariant is a T-vector for which $\underline{C}\cdot\underline{t}=\underline{0}$ holds. A subnet of the underlying net with incidenc matrix $\underline{C}$ is denotated as the graphical representation of a semi-positive invariant $\underline{t}$ iff it contains all transitions t_i which correspond with positive components c_x in the invariant $\underline{t}$ (with $i=x$) and all places/edges which are incident/adjacent to these transitions.

Since the firing rule FR of place/transition-nets defines a follower marking $\underline{M}'$ of a reference marking $\underline{M}$ by $\underline{M}'=\underline{M}+\underline{C}\cdot\underline{t}$, a semi-positive T-invariant may be interpreted as the firing vector of a firing sequence which reproduces the reference marking because of $\underline{M}'=\underline{M}+\underline{0}=\underline{M}$. However, it must be stressed that this interpretation has to be justified by applying the firing sequence which agrees with the firing vector $\underline{t}$ to the reference

22 Z denotes the set of integers.

marking $\underline{M}$[23]. Because the fulfilment of the equality mentioned above only reflects the effect of transtion firing, the activation-predicates have not been considered. Therefore a firing sequence with a T-invariant $\underline{t}$ as firing vector, which seems to reproduce the reference marking, may be inadmissible since at least one of the "fired" transitions is not activated under the marking that is produced by carrying out the firing sequence up to this transition.

P-invariants are complements of T-invariants. A P-invariant is defined as a P-vector which fulfils the equality $\underline{p}^{tr} \cdot \underline{C} = \underline{0}^{tr}$. Such P-invariants express properties of net models that hold "invariant" with respect to varying net markings. Those characteristic structural problem aspects can be used for problem analysis. But the interpretation of the "real meaning" of P-invariants often causes difficulties that arise when there are no strong connections between the formal components of the underlying P-vector on the one side and "relevant" aspects of the modelled real problem on the other side. Therefore P-invariants are not the main topic of this article. They serve only for defining the following net theorem.

Lautenbach (1985) has formulated and proved a theorem[24] which holds for every net model of problem aspects stated in propositional logic as far as three conditions are fulfilled. Firstly the model must be finite[25]. Secondly finite token capacities of places do

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23 It is possible that more than one firing sequence agrees with the firing vector given by a T-invariant $\underline{t}$. This happens when several firing sequences possess the equal firing-numbers for each transition but differ with respect to the ordering of firing acts.

24 See also Fidelak (1986a) and Fidelak (1986b).

25 This does not restrict the practical importance of the net theorem, since every problem description that is formulated with the help of finitely many propositions - i.g. conjunctive connected clauses - leads to a finite net model as its representation.

not exisist[26]. Thirdly only pure nets are considered[27]. According to the generalized construction of net models each transition of an analyzed net is considered - together with its incident places for the involved atomar propositions[28] - as a clause. Therefore a net model represents a finte set of clauses that are connected in a conjunctive manner and completely describe the logical aspects of a problem.

Under these assumptions the net theorem states that the set of clauses is inconsistent iff:

- there exists a semi-positive T-invariant t such that at least one firing sequence with t as firing vector really reproduces the zero-marking of the net and
- the subnet which represents the forementioned T-invariant does not contain any non-trivial P-invariant[29].

If a contradiction in the set of clauses has been identified it cannot be resolved by changing the actual problem situation, because no concrete marking of the net model is object of the net theorem. Therefore the contradiction indicates a structural inconsistency in the propositional description of logical problem aspects. It is impossible to construct any marking of the net model that would remove the contradiction. Hence there cannot exist any consistent problem situation represented by a net marking.

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26 This aspect will be discussed at the end of this chapter.

27 Impure nets can be augmented to pure nets by refining the transitions which are involved in 1-loops. Each such transition is replaced ba a sequence of two transitions which enclose an additional place; see Lautenbach (1987).

28 The mentioning of the incident places is omitted in the following if no misunderstandings are to be expected.

29 If such a subnet exists it is caused by a vicious circle that occurs during the reproducing of the zero-marking by that firing sequence which has the analyzed T-invariant t as firing vector; see Fidelak (1986b).

In addition the source of a detected structural inconsistency can be identified. The contradiction is caused by that subset of clauses being represented in the net model by transitions which correspond with positive components in the T-invariant t fulfilling the net theorem. It is impossible to assign any combination of truth values to the atomar propositions, which are the constituents of the clauses, in such a manner that the conjunction of all clauses becomes true. Therefore the complex proposition in conjunctive normal form which is represented by the whole net model must be false in any case. This meta-proposition is equivalent with the recognition that the modelled problem description contains a structural inconsistency caused by the set of clauses defined above.

For the purpose of identifying T-invariants with the special properties mentioned above it is necessary to generate all T-invariants of a given net model and to evaluate those invariants whether they fulfil the conditions of the net theorem. The second step of evaluating generated T-invariants involves an additional invariant analysis, namely the generation - and evaluation - of P-invariants of the representing subnets. The task of generating T- or P-invariants causes the main difficulties in applying the net theorem.

T- and P-invariants are integer solutions of the linear-homogenous equation systems $\underline{C} \cdot \underline{t} = \underline{0}$ and $\underline{p}^{tr} \cdot \underline{C} = \underline{0}^{tr}$, respectively. The problem to generate solutions of such diophantic equation systems can be solved in principle by means of linear algebra. This reveals once again the potential of net theory to combine a compact and transparent graphical problem description with the applicability of powerful algebraic techniques. These techniques allow to analyze logical description properties on the base of consistency[30].

The generation of net invariants is supported by a wide range of software tools for Petri net analysis based on algebraic techniques; see the tool-overview in Feldbrugge (1987). But these programs do usually not guarantee to generate all theoretically existent invariants. Therefore the contribution of Pascoletti (1985) deserves special attention. It is based on sophisticated algebraic anlaysis and enables to identify all "simple" T- and P-invariants of nets. These invariants are "simple" in the sense that each theoretically existent invariant can be build up by linear combination of simple invariants. Hence the theoretical problem of generating all invariants of a net - and consequently the detection of all contradictions in net-based descriptions of logical problem aspects, too - is solved in principle. The two distinct algebraic algorithms developed by Pascoletti can be augmented by combinatorial algorithms to generate and evaluate all invariants of a net model as far as it is necessary for applying the forementioned net theorem. Therefore the analytical requirement of chapter 1 concerning the possibility of computerized model analysis is fulfilled.

However, it remains the practical problem that the bulding up of all invariants and their evaluation may combinatorically explode. Although the automatical execution of generating and evaluating invariants can diminish the practical importance of combinatorial explosion, this difficulty remains in principle. This

30 Another fundamental aspect of net theory concerns the combination of linear algebraic techniques applied to net models on the one side with the artificial intelligence technique of proving theorems based on the refutation principle on the other side. This connection between linear algebra and artificial intelligence is constituted by the prove of Lautenbach's net theorem which grounds on the refutation principle. But this bridging to artificial intelligence lies beyond the scope of this article; see Lautenbach (1985), Fidelak (1986a), Zelewski (1986).

holds at least for such nets in which the number of invariants considerably exceeds the number of simple invariants. But at this time the proportion between these both numbers of invariants is not yet well explored, so that judges about the practical importance of combinatorial explosion are problematic in the context of the net theorem.

The abstraction from finite token capacities in Lautenbach's net theorem causes a serious drawback of invariant analysis. Nets with unbounded token capacities for their places may allow some firing sequences which are inadmissible in complemantary nets with same structures but finite token capacities. Such a deviation of possible net behaviors plays a role for net analysis of logical problem descriptions, because nets with token capacity $K(p_j)=1$ for all places $p_j \varepsilon P$ are required (see chapter 2.1).

There are constructions that allow to transform nets with finite token capacities into nets with unbounded token capacities with the help of additional complementary places[31]. But these complementary places adversely affect the transparency and compactness of net models. Furthermore it is assumed that the sum of tokens on a place with finite token capacity and its complementary place must equal to the token capacity under each reachable marking. This assumption contradicts the zero-marking of the invariant analysis in the special form of the net theorem. Therefore a contradiction or the consistency of a net model which have been "proved" with the help of the net theorem must be critically analyzed whether the "prove" rests either on the structure of the logical problem description or on

31 See e.g. Lautenbach (1987).

the abstraction from finite token capacities[32]. The expression "prove" is used in this restricted sense in the following.

For the remaining discussions of this chapter it is supposed that a net model of logical problem aspects is proved to be structurally consistent. Then the original net model can be augmented in a manner that enables to identify all possible situative inconsistencies. These contradictions depend on meta-propositions about the truth values of those atomar (object-)propositions which constitute the modelled problem description. For this purpose an atomar proposition is appended as an atomar clause (literal) to the original problem description iff it is contained in at least one composite proposition of the problem description but does not form an atomar clause of the description's conjunctive normal form. According to this the negation of each atomar proposition out of the original problem description is appended as an atomar clause (literal) iff this negation is not yet an atomar clause of the description.

The resulting augmented net must be inconsistent since it includes conjunctions of atomar propositions and their negations. The T-invariants are neglected which fulfil the net theorem and are caused by those sets of clauses that consist of a atomar proposition and its negation. Then the remaining T-invariants which also satisfy the conditions of the net theorem must indicate all possible situative inconsistencies caused by assigment of truth values to the atomar proposi-

32 This anlayse can be carried out by studying the firing sequences which both agree with the firing vector of the T-invariant and reproduce the zero-marking. Iff such a firing sequence exceeds at least once the token capacity $K(p_j)=1$ of any place p_j then the detected contradiction is not necessarily caused by the problem description but may be effected by the abstraction from token capacities.

tions[33]. This is implied by the premise that the original net model was proven to be structurally consistent. Because of this premise any detected new "structural" inconsistency in the augmented net model must be caused by the subset of those transitions which both correspond to positive components in one of the forementioned remaining T-invariants and represent appended (negations of) atomar propositions. Only these atomar propositions are considered in the following. They gain the truth values "true" or "false" iff they are appended to the original problem description as atomar propositions or as negations of atomar propositions, respectively. It can be shown that this combination of truth values forms a meta-proposition about an inconsistent problem situation[34], i.e. an inconsistent marking of the net model which could be detected with the help of fact-nets, too. Hence the set of all remaining T-invariants allows to determine all net markings which represent inconsistent descriptions of problem aspects that are marking-dependent, that is situative.

This way of identifying all possible inconsistent problem situations in a net model that is proved to be

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33 If there exist no remaining T-invariants the net model represents a tautology which is true with respect to all possible problem situations. This special case is ruled out in the following, because empirically meaningful problem descriptions are not tautological.

34 The underlying idea is the following: The conjunction of the clauses which are implied by each remaining T-invariant is a logical contradiction, i.e. it must be false under every possible combination of truth values for the atomar propositions of its clauses. Since the original net model is structurally consistent, the contradiction must be caused by the appended atomar propositions and negations of atomar propositions. There is exact one combination under which the conjunction of these appended literals would be true. It is that combination which assigns the truth value "true" to each literal. Therefore appended atomar propositions are true and appended negations of atomar propositions are false with respect to this assignment. But the truth of the conjunction of appended literals contradicts the proved inconsistency of the augmentioned net model. Hence this combination of truth values cannot be consistent with the clauses of the original net model which was proved to be structurally consistent.

structurally consistent can be used to describe the abstract space of logically consistent problem solutions in a net-based, but algebraic formulated manner. It is possible to combine this algebraic defined logical view of solution space with a conventional algebraic OR-model which represents the non-logical aspects of a problem description. The resulting composed algebraic model may be solved by conventional OR-techniques. However, this approach is not considered in more detail[35] since the solution of models does not concern the scope of this article which is concentrated on the construction of models. Furthermore the question had to be answered whether the net-based approach of modelling logical problem aspects leads to easier solvable models than the conventional use of logic variables[36].

4 Examples for Net-Based Analysis of Logical Problem Descriptions

4.1 Some Simple Theoretical Examples for Demonstrating Structural and Situative Inconsistencies

The following examples only serve to illustrate the theoretical concepts of situative and structural analysis concerning propositional descriptions of logical problem aspects. They are reduced to very simple nets without any reference to real problems in order to elucidate the potential of possible findings. In accordance to the elementary net structure the modelled logical aspects are trivial; in chapter 4.2 an example will be discussed which is a little bit more substantial.

The following nets represent composed propositions which are expressed in conjunctive normal form, i.e. as conjunctions of clauses. Each clause C_i is represented by a transition t_i with its incident places. Each of

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35 It is exhaustively outlined in Zelewski (1986).

36 A remark to this aspect is containd in Zelewski (1986).

these places p_j is characteristic for an atomar proposition P_j constituting the clause C_i. The markings $M(p_j)=1$ and $M(p_j)=0$ of a place p_j indicate that the corresponding proposition P_j is true or false, respectively. The truth value of the clause follows from the markings of its incident places, i.e. from the truth values of its constituting atomar propositions. According to the work of Pascoletti only the simple T-invariants are considered, but not the T-invariants which are linear combinations of simple T-invariants.

Fig. 2 shows the simplest structural inconsistency which is logically possible. The composed proposition $P_1 \wedge (\neg P_1)$ that simultaneously states an atomar proposition P_1 and its negation $\neg P_1$ must be false under any possible assignment of truth values to its atomar proposition P_1. Correspondingly the T-invariant $\underline{t}_2$ fulfils the forementioned conditions of the net theorem. This invariant is the firing vector of the firing sequence $\langle t_1, t_2 \rangle$ which reproduces the zero-marking. The components $c_1=1$ and $c_2=1$ of the invariant $\underline{t}_2$ indicate that the set $\{C_1, C_2\}$ of clauses is structurally inconsistent.

$\underline{C}_2 = \begin{vmatrix} 1 & -1 \end{vmatrix}$ $\underline{t}_2 = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$

$\underline{C}_2 \cdot \underline{t}_2 = \underline{0} = 0$

Fig. 2: proposition $P_1 \wedge (\neg P_1) \Leftrightarrow C_1 \wedge C_2$
with $C_1 \Leftrightarrow P_1$, $C_2 \Leftrightarrow \neg P_1$

The net model of fig. 3 is structural consistent since there exist no semi-positive T-invariants reproducing the zero-marking. But the situative problem description which assigns the truth values "true" and "false" to the atomar propositions P_1 and P_2, respectively, is inconsistent. The correspondingly marked net with $M(p_1)=1$ and $M(p_2)=0$ contains the activated transition t_1 which contradicts the requirement of dead transitions in fact-nets. Since the crucial transition t_1

represents the composed clause $C_1 \Leftrightarrow P_1 \rightarrow P_2$ it follows that the supposed assignment of truth values to the atomar propositions is not consistent with the subjunction of clause C_1.

$$\underline{C}_3 = \begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

$$\underline{t}_3 = \begin{vmatrix} 1 \\ 1 \\ -1 \end{vmatrix} \text{ and others}$$

$\underline{C}_3 \cdot \underline{t}_3 = \underline{0}$, but: $\underline{t}_3$ is not semi-positive

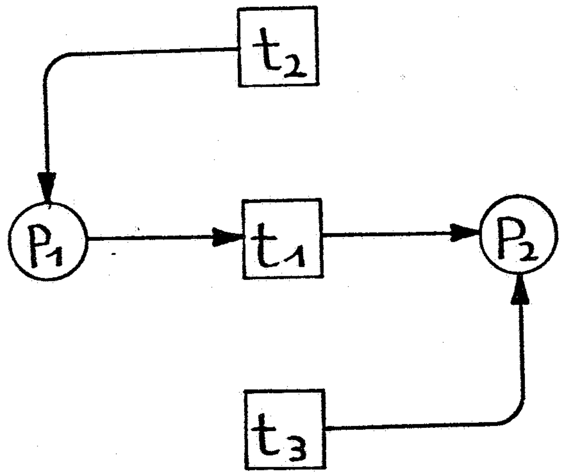

Fig. 3: proposition $(P_1 \rightarrow P_2) \wedge P_1 \wedge P_2 \Leftrightarrow C_1 \wedge C_2 \wedge C_3$ with $C_1 \Leftrightarrow (\neg P_1) \vee P_2$, $C_2 \Leftrightarrow P_1$, $C_3 \Leftrightarrow P_2$

Fig. 4 differs from fig. 3 only with regard to the clause $C_3 \Leftrightarrow \neg P_2$ which is replaced for $C_3 \Leftrightarrow P_2$. This little change causes an important logical consequence. The net of fig. 4 models a structurally inconsistent problem description. The T-invariant $\underline{t}_4$ satisfies the requirements of the net theorem. It is the firing vector of the firing sequence $\langle t_2, t_1, t_3 \rangle$ which reproduces the zero-marking. Hence it cannot exist any problem situation represented by a net marking which is logically consistent. This result is evident, since the modelled proposition contradicts the wellknown inference rule of modus ponens.

$$\underline{C}_4 = \begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \end{vmatrix}$$

$$\underline{t}_4 = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}$$

$\underline{C}_4 \cdot \underline{t}_4 = \underline{0}$

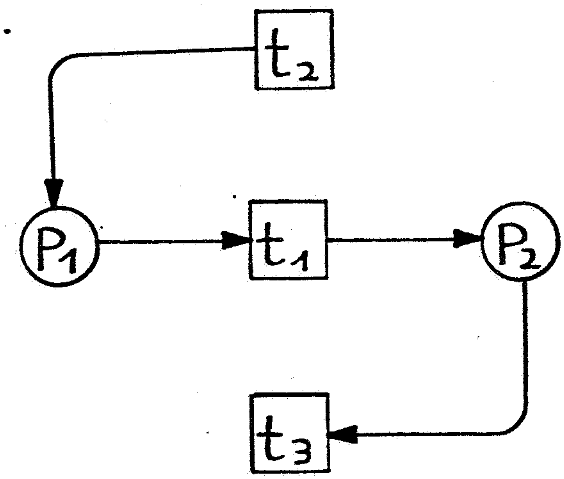

Fig. 4: proposition $(P_1 \rightarrow P_2) \wedge P_1 \wedge (\neg P_2) \Leftrightarrow C_1 \wedge C_2 \wedge C_3$ with $C_1 \Leftrightarrow (\neg P_1) \vee P_2$, $C_2 \Leftrightarrow P_1$, $C_3 \Leftrightarrow \neg P_2$

The findings of fig. 3 and fig. 4 are integrated in fig. 5 which is the augmention of the composed clause $C_1 <=> P_1 -> P_2$ with respect to its atomar propositions and their negations. The net of fig. 5 possesses the trivial structural inconsistencies between each atomar proposition and its negation correspondingly to fig. 1. They are represented by the T-invariants $\underline{t}_{5.1}$ and $\underline{t}_{5.2}$. But there remains a third T-invariant $\underline{t}_{5.3}$, which is not trivial. Its structural inconsistency is caused by the set $\{C_1, C_2, C_5\}$ of clauses in accordance to its components with positive values. It shows that the truth of clauses C_2 and C_5 - i.e. atomar propositions P_1 and P_2 are true and false, respectively, - cannot be consistent with the truth of the subjunction of clause C_1. This is exact the forementioned case of situative inconsistency in fig. 3 and of structural inconsistency in fig. 4. Since the net of fig. 5 contains no more remaining T-invariants it can be concluded that the inconsistency detected by invariant $\underline{t}_{5.3}$ is the only non-trivial inconsistency which may occur in relation to the subjunction of clause $C_1 <=> P_1 -> P_2$. Whether this inconsistency is recognized as being situative or structural depends on the structure of the propositional problem description. The structure underlying the net of fig. 3 led to a situative, the structure belonging to the nets of fig. 4 and 5 to a structural inconsistency.

$$\underline{C}_5 = \begin{vmatrix} -1 & 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & -1 \end{vmatrix}$$

$$\underline{t}_{5.1} = \begin{vmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{vmatrix} \quad \underline{t}_{5.2} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{vmatrix} \quad \underline{t}_{5.3} = \begin{vmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{vmatrix}$$

$$\underline{C}_5 \cdot \underline{t}_{5.1} = \underline{C}_5 \cdot \underline{t}_{5.2} = \underline{C}_5 \cdot \underline{t}_{5.3} = \underline{0}$$

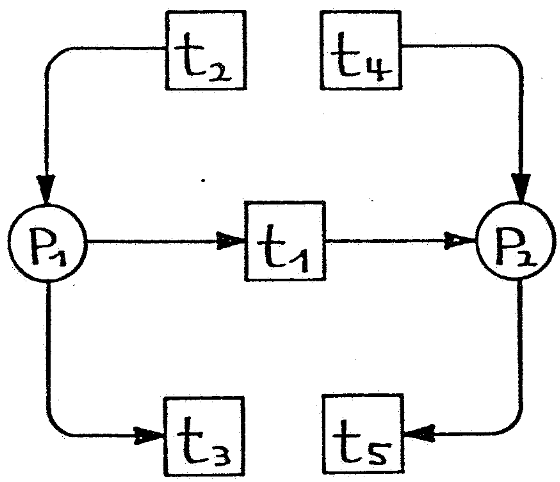

Fig. 5: proposition $(P_1 \rightarrow P_2) \wedge P_1 \wedge (\neg P_1) \wedge P_2 \wedge (\neg P_2) <=> \ldots$
$C_1 \wedge C_2 \wedge C_3 \wedge C_4 \wedge C_5$ with $C_1 <=> (\neg P_1) \vee P_2$, $C_2 <=> P_1$, $C_3 <=> \neg P_1$, $C_4 <=> P_2$, $C_5 <=> \neg P_2$

The net of fig. 6 is structural consistent. It contains a semi-positive T-invariant $\underline{t}_6$ which defines the firing vector of the firing sequences $\langle t_1, t_2 \rangle$ and $\langle t_2, t_1 \rangle$. However, these firing sequences are not able to reproduce the zero-marking since neither transition t_1 nor transition t_2 is activated under this marking[37]. Hence the net theorem is not fulfilled. But the modelling of a situative problem description in which the atomar propositions P_1 and P_2 are false (true) and true (false), respectively, is inconsistent. Under the corresponding marking $\underline{M}$ with $M(p_1)=0$ and $M(p_2)=1$ ($M(p_1)=1$ and $M(p_2)=0$) the transition t_2 (t_1) is activated. This contradicts the requirement of dedad transtions in fact-nets.

$$\underline{C}_6 = \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} \qquad \underline{t}_6 = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$$

$\underline{C}_6 \cdot \underline{t}_6 = \underline{0}$, but: $\underline{t}_6$ does not define the firing vector of a zero-marking reproducing firing sequence

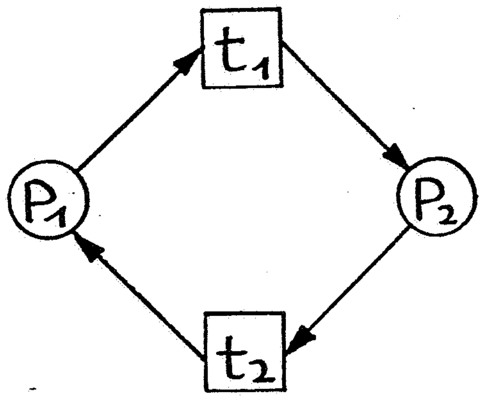

Fig. 6: proposition $(P_1 \rightarrow P_2) \wedge (P_2 \rightarrow P_1) \Leftrightarrow C_1 \wedge C_2$ with $C_1 \Leftrightarrow (\neg P_1) \vee P_2$, $C_2 \Leftrightarrow (\neg P_2) \vee P_1$

The net of fig. 7 is structural consistent; its composed proposition is logically equivalent to the disjunction of the atomar propositions P_1 and P_2. It contains a semi-positive T-invariant $\underline{t}_7$ which defines the firing vector of the firing sequence $\langle t_2, t_1 \rangle$ which reproduces the zero-marking[38]. However this invariant does not fulfil the second condition of the net theorem. The "subnet" of the graphical representation of the invariant $\underline{t}_7$ is identical with the whole net. This

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37 The same holds for all other - infinetely many - semi-positive T-invariants $\underline{t}_{6.k}^{tr}=(k,k)$ with $k=2,3,...$, which are degenerated linear "combinations" of the simple T-invariant $\underline{t}_6$. Furthermore there exist also infinetely many T-invariants which are not semi-positive, e.g. $\underline{t}_{6'}^{tr}=(-1,-1)$.

38 Once again there exist infinitely many not-simple semi-positive T-invariants $\underline{t}_{7.k}^{tr}=(k,k)$ with $k=2,3,...$

"subnet" contains two non-trivial P-invariants $\underline{p}_{7.1}$ and $\underline{p}_{7.2}$ both contradicting the net theorem[39].

$\underline{C}_7 = \begin{vmatrix} -1 & 1 \\ -1 & 1 \end{vmatrix} \qquad \underline{t}_7 = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$

$\underline{C}_7 \cdot \underline{t}_7 = \underline{0}$, but: there exist non-trivial P-invariants with:

$\underline{p}_{7.1} = \begin{vmatrix} 1 \\ -1 \end{vmatrix} \qquad \underline{p}_{7.2} = \begin{vmatrix} -1 \\ 1 \end{vmatrix}$

$\underline{p}_{7.1}^{tr} \cdot \underline{C}_7 = \underline{p}_{7.2}^{tr} \cdot \underline{C}_7 = \underline{0}^{tr}$

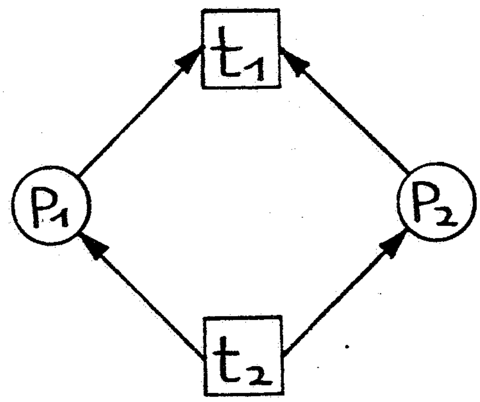

Fig. 7: proposition $(P_1 \rightarrow (\neg P_2)) \wedge (P_1 \vee P_2) \Leftrightarrow C_1 \wedge C_2$ with $C_1 \Leftrightarrow (\neg P_1) \vee (\neg P_2)$, $C_2 \Leftrightarrow P_1 \vee P_2$

4.2 An Application-Oriented Example Based on Modelling the Design of Balance Sheets

Johänntgen-Holthoff (1986) proposed a complex decision model concerning the practice of balance sheet make-up. The model is constructed as an mixed-integer linear program with the help of conventional OR-techniques. In the following only a part of this voluminous model is discussed. This part models the logical aspects which must be considered in order to design the taxable base of corporate income tax with regard to loss carryback and carryforward[40]. The logical aspects are expressed by binary logic variables. Since this (partial) model remains to be very complex – it fills up 24 pages by Johänntgen-Holthoff (1986) – in the following only non-

39 Infinetely many not-simple not-trivial P-invariants can be derived from this two simple non-trivial P-invariants, e.g. $\underline{p}_{7.3}^{tr} = (2,-2)$.

40 The design of balance sheets bases on German corporation income tax law (KStG), corporation income tax regulations (KStR) and income tax law (EStG), namely § 8/4 KStG and § 37/2 KStR in connection with § 10d EStG.

negative taxable bases without complications caused by loss carrybacks or carryforwards are considered.

The conventional, voluminous and intransparent model of Johänntgen-Holthoff - which is referenced as "original model" - is reformulated as a compact net model. The naming, indexing and numbering of the variables and formulas are adopted from the original model in order to support comparisons between the modelling alternatives[41].

The original model involves 16 atomar propositions P_j with j=1,...,9,11,...17[42] relating to a reference year indexed by "t". They are listed in the following:

P_1 : "There is no loss $V_{t/-}$ with relevance to corporate income tax: $V_{t/-}=0$."

P_2 : "The profit G_t with relevance to corporate income tax equals the taxable base $G_{t/0}$ before considering loss carryback or carryforward: $G_t=G_{t/0}$."

P_3 : "There is no loss carryback $X_{t/-2}$ from two years ago: $X_{t/-2}=0$."

P_4 : "There is no loss carryback $X_{t/-1}$ from the previous year: $X_{t/-1}=0$."

P_5 : "The profit $G_{t/B}$ after deduction of the cumulated loss carrybacks of the previous years equals the difference between the profit G_t and the non-negative loss carryforwards Z_t of the previous years: $G_{t/B}=G_t-Z_t$."

P_6 : "In the next year no loss carryforward is showed: $Z_{t+1}=0$."

P_7 : "§ 10d sentence 1 EStG is applied for determing taxable base E_t of corporate income tax: $E_t=G_{t/B}-X_{t+2/-2}-X_{t+1/-1}$."

P_8 : "The amount $G_{t/E}$ limiting the loss carryback pursuant to § 8/4 KStG equals zero: $G_{t/E}=0$."

P_9 : "The amount $G_{t/E}$ limiting the loss carryback pursuant to § 8/4 KStG equals the difference between the profit $G_{t/B}$ after deduction of the cumulated loss carrybacks of the previous years and the gross distribution of dividends $X_{a,t/b}$: $G_{t/E}=G_{t/B}-X_{a,t/b}$."

P_{11} : "The taxable base E_t of corporate income tax pursuant to § 10d sentence 1 EStG equals zero: $E_t=0$."

41 For a more detailed explanation of the following formulas see Zelewski (1986).

42 The proposition P_{10} of the original model is omitted here since it is the same as proposition P_8.

P_{12}: "The loss carryforward Z_{t+1} of the previous years which is reduced by the profit G_t is not corrected for the next year: $Z_{t+1}=Z_t-G_t$."

P_{13}: "The loss carryforward Z_{t+1} of the previous years which is reduced by the profit G_t is corrected for the next year by loss carryforwards which are no more deductable: $Z_{t+1}=Z_t-G_t-(Z_{t-4}-G_t-G_{t-1}-G_{t-2}-G_{t-3}-G_{t-4})$."

P_{14}: "The taxable base $G_{t/0}$ of corporate income tax before considering loss carrybacks or carryforwards is not negative: $G_{t/0}\geq 0$."

P_{15}: "The profit G_t with relevance to corporate income tax is not less than the cumulated loss carryforward Z_t of the previous years: $G_t\geq Z_t$."

P_{16}: "The profit $G_{t/B}$ after deduction of the cumulated loss carrybacks of the previous years does not cover the gross distribution of dividends $X_{a,t/b}$: $G_{t/B}<X_{a,t/b}$."

P_{17}: "The loss carryforward Z_{t-4} of year t-4 has been compensated by profits up to the reference year t: $Z_{t-4}\leq G_t+G_{t-1}+G_{t-2}+G_{t-3}+G_{t-4}$."

With the help of these atomar propositions the logigal requirements which must be fulfilled by any legally admissible design of balance sheets are stated as 8 composed propositions P_j with j=18,...,25[43] which are supposed to be conjunctively connected. These propositions are composed as subjunctions so that they appear in the wellknown form of "decision" rules. In order to transform this propositional description of logical problem aspects into an equivalent net model each subjunction is reformulated in conjunctive normal form with the help of clauses C_i (i=1,...,13):

P_{18}: $P_{14} \rightarrow (P_1 \wedge P_2 \wedge P_3 \wedge P_4) \Leftrightarrow C_1 \wedge C_2 \wedge C_3 \wedge C_4$

with: $C_1 \Leftrightarrow (\neg P_{14}) \vee P_1$
$C_2 \Leftrightarrow (\neg P_{14}) \vee P_2$
$C_3 \Leftrightarrow (\neg P_{14}) \vee P_3$
$C_4 \Leftrightarrow (\neg P_{14}) \vee P_4$

P_{19}: $P_{15} \rightarrow (P_5 \wedge P_6) \Leftrightarrow C_5 \wedge C_6$

with: $C_5 \Leftrightarrow (\neg P_{15}) \vee P_5$
$C_6 \Leftrightarrow (\neg P_{15}) \vee P_6$

P_{20}: $(P_{14} \wedge P_{15}) \rightarrow P_7 \Leftrightarrow C_7$

with: $C_7 \Leftrightarrow (\neg P_{14}) \vee (\neg P_{15}) \vee P_7$

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43 This indexing cannot be found by Johänntgen-Holthoff, because the formulas used there do not follow the strict propositional approach of this article.

P_{21}: $(P_{14} \wedge P_{15} \wedge P_{16}) \rightarrow P_8 \iff C_8$

with: $C_8 \iff (\neg P_{14}) \vee (\neg P_{15}) \vee (\neg P_{16}) \vee P_8$

P_{22}: $(P_{14} \wedge P_{15} \wedge (\neg P_{16})) \rightarrow P_9 \iff C_9$

with: $C_9 \iff (\neg P_{14}) \vee (\neg P_{15}) \vee P_9 \vee P_{16}$

P_{23}: $(P_{14} \wedge (\neg P_{15})) \rightarrow (P_8 \wedge P_{11}) \iff C_{10} \wedge C_{11}$

with: $C_{10} \iff (\neg P_{14}) \vee P_8 \vee P_{15}$
$C_{11} \iff (\neg P_{14}) \vee P_{11} \vee P_{15}$

P_{24}: $(P_{14} \wedge (\neg P_{15}) \wedge P_{17}) \rightarrow P_{12} \iff C_{12}$

with: $C_{12} \iff (\neg P_{14}) \vee (\neg P_{17}) \vee P_{12} \vee P_{15}$

P_{25}: $(P_{14} \wedge (\neg P_{15}) \wedge (\neg P_{17})) \rightarrow P_{13} \iff C_{13}$

with: $C_{13} \iff (\neg P_{14}) \vee P_{13} \vee P_{15} \vee P_{17}$

The net model of Fig. 8 represents the logical problem description given by the subjunctions $P_{18},\dots,P_{25}$. Each place p_j of the net represents an atomar proposition P_j with the same index which is a component of the forementioned subjunctions. Each transition t_i of the net represents a clause C_i with the same index which is a component of the conjunctive normal forms of the subjunctions. The whole net model represents a conjunction of all these clauses.

The net model of fig. 8 contains no T-invariant which fulfils the net theorem. Therefore the logical problem description is structurally consistent. But there are several inconsistent problem situations defined by assignments of truth values to the atomar propositions $P_1,\dots,P_{17}$. For example all problem situations are inconsistent which suppose that the propositions P_{14}, P_{15} and P_{16} are true and the proposition P_8 is false. The inconsistency of such meta-propositions is not at all obvious as it can be seen by comparing them with the above listed definitions of the involved propositions.

This inconsistencies can be proved by applying the fact-net based approach. The supposed assignments of truth values are represented by markings $\underline{M}$ with $M(p_{14})=M(p_{15})=M(p_{16})=1$ and $M(p_8)=0$. Under such markings the transition t_8 is activated. This contradicts the

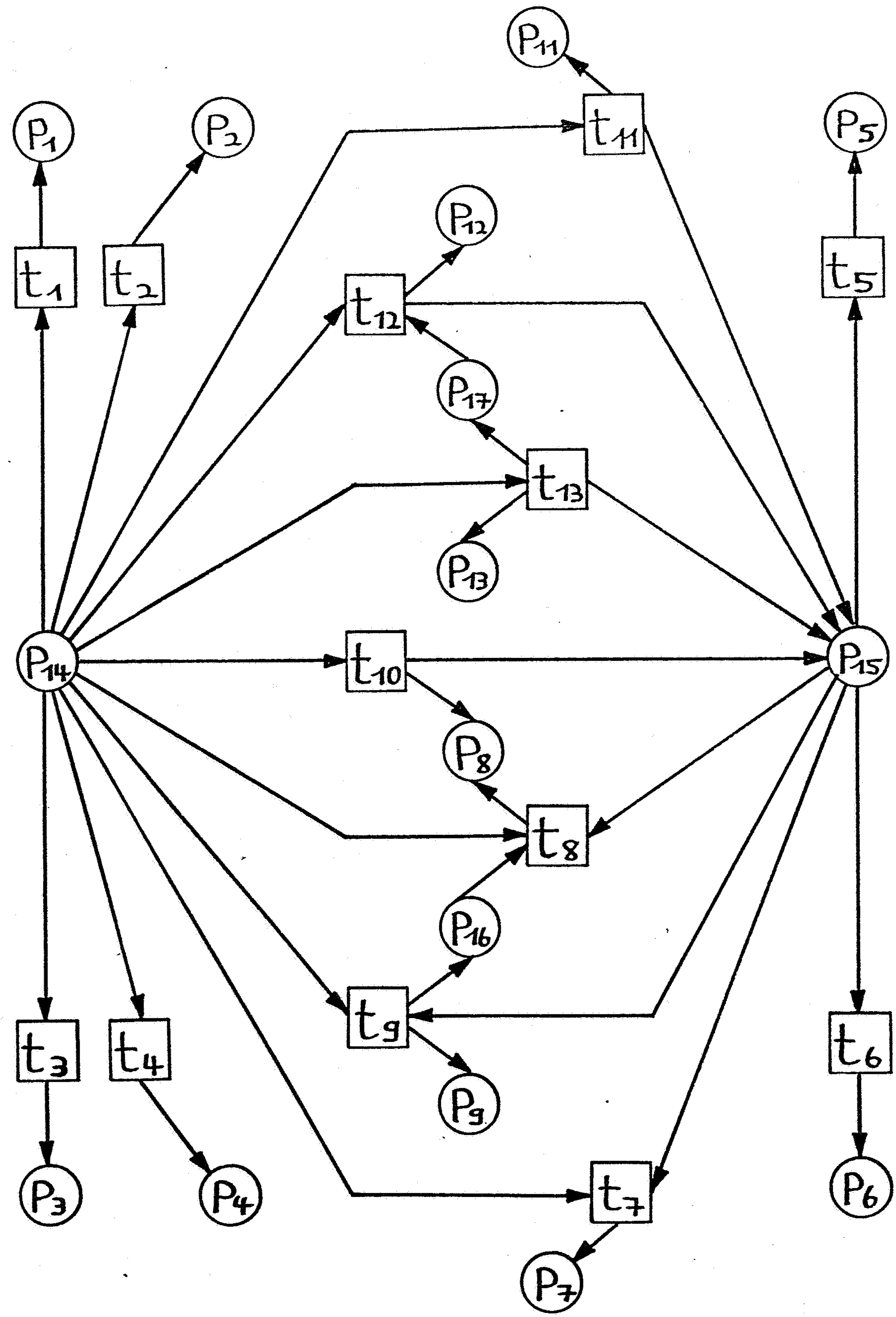

Fig. 8: net model of the description of logical aspects concerning the problem of balance sheet make-up

requirement of dead transitions in fact-nets. Therefore the markings $\underline{M}$ must represent inconsistent descriptions of problem situations.

The inconsistency to suppose that the propositions P_{14}, P_{15} and P_{16} are true and the proposition P_8 is false can be proved by the approach of net augmentioning, too. Fig. 9 represents the augmented net model which is derived from the original problem description - modelled by the net of fig. 8 - by appending the underlying atomar propositions $P_1,\dots,P_{17}$ and their negations as atomar clauses $C_{14},\dots,C_{45}$, which are represented by the transitions $t_{14},\dots,t_{45}$ and their incident places. Fig. 10 shows the incidence matrix $\underline{C}_9$ of the augmentioned net model.

For the augmentioned net holds - among others - the T-invariant $\underline{t}_9$ which satisfies all conditions of the net theorem. This invariant is defined by:

$$\underline{t}_9^{tr} = (c_x \mid c_x=1 \text{ for } x=8,29,38,40,42 \text{ and } c_x=0 \text{ otherwise})$$

The invariant fulfils the equality $\underline{C}_9 \cdot \underline{t}_9 = \underline{0}$ and is the firing vector of several firing sequences each of which reproduces the zero-marking. For example, $\langle t_{38}, t_{40}, t_{42}, t_8, t_{29} \rangle$ is such a firing sequence. Therefore the set $\{C_{38}, C_{40}, C_{42}, C_8, C_{29}\}$ of clauses represents (a part of) a structurally inconsistent problem description. This implies - as explained in chapter 3.2 - that the truth of the atomar clauses C_{29}, C_{38}, C_{40} and C_{42} cannot be consistent with the composed clause C_8. The forementioned atomar clauses are equivalent to the propositions $\neg P_8$, P_{14}, P_{15} and P_{16}, respectively. Because the supposed truth of the atomar clauses is inconsistent, it must be inconsistent to suppose that the propositions P_{14}, P_{15} and P_{16} are true and the proposition P_8 is false. This is just the inconsistency detected above by the fact-net based approach.

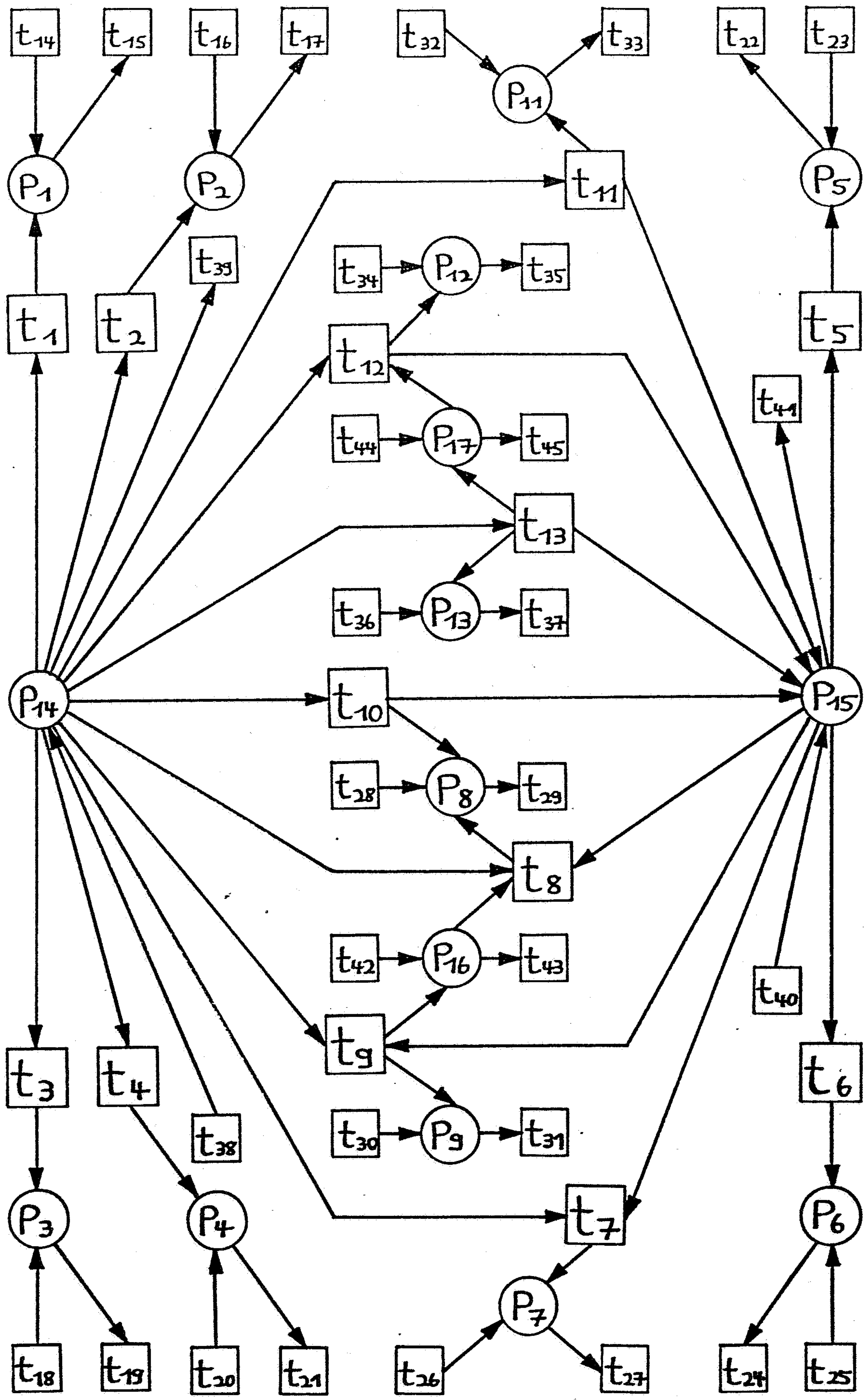

Fig. 9: augmentioned net model of fig. 8

$$\underline{C}_9{}^{tr} = \begin{vmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{vmatrix}$$

Fig. 10: transposed incidence matrix $\underline{C}_9$ for the net model of fig. 9

5 Conclusion

The net-based modelling of logical problem aspects has two main advantages. Firstly the graphical representation of net models makes it easier to communicate about such logical aspects. Secondly the logical analysis can be reduced to algebraic analysis which allows the application of powerful analytic algorithms and their implementations in computer software. Net theory constitutes the pivotal point: it establishes a graphical, well understandable and communicable representation on the one side and possesses a sophisticated mathematical theory with strong connections to linear algebra on the other side. Real problems can be analyzed on the base of proving (in)consistencies in the same manner as it has been discussed in chapter 4 with regard to some illustrative examples.

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